

文章编号: 1000-8055(2009)02-0269-08

# Thermodynamic cycle analysis of solid propellant air turbo rocket

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**Abstract:** Solid propellant air turbo rocket (SPATR) is an air breathing propulsion system. A numerical model of performance and characteristics analysis for SPATR was presented and the corresponding computer program was written according to the operation characteristics of SPATR. The influence on the SPATR performance at design point caused by the gas generator exit parameters and compressor pressure ratio had been computed and analyzed in detail. The off design performance of SPATR at sea level and high altitude had also been computed. The performance of thrust and specific impulse for SPATR with different solid propellant had been compared at off design points, and the off design performance comparison had been made between fuel rich and oxygen rich. The computation results indicated that SPATR operates within wide range of Mach number (0 ~ 3) and altitude (0~ 12 km), and SPATR possesses high specific thrust (1 200 N/(kg/s)) and high specific impulse (7 000 N/(kg/s)) when fuel air ratio of combustor equals fuel air ratio.

**Key words:** solid propellant air turbo rocket (SPATR); off design points; numerical model; fuel rich gases; solid propellant

**CLC number:** V435      **Document code:** A

|                |                                    |                               |                                          |
|----------------|------------------------------------|-------------------------------|------------------------------------------|
| <b>Symbols</b> |                                    | $L_o$                         | proper mixing ratio                      |
| $T$            | total temperature                  | <b>Subscripts and station</b> |                                          |
| $p$            | total pressure                     | B                             | combustor                                |
| $\pi$          | pressure ratio                     | C                             | compressor                               |
| $\eta$         | efficiency                         | F                             | fuel rich gases                          |
| $c_p$          | specific heat at constant pressure | T                             | turbine                                  |
| K              | ratio of specific heats            | a                             | air                                      |
| $L_c$          | compressor power                   | g                             | mixed gases of the fuel rich and the air |
| $L_T$          | turbine power                      | t                             | total parameters                         |
| $W$            | Mass flow                          | 0                             | far upstream or free stream              |
| $f$            | fuel air ratio                     | 2                             | compressor inlet                         |
| $f_o$          | equivalent fuel air ratio          | 4                             | turbine inlet                            |
| $H$            | total enthalpy                     | 5I                            | turbine exit                             |
| $F_n$          | thrust                             | 5II                           | compressor exit                          |
| $F_s$          | specific thrust                    | 6                             | combustor inlet                          |
| $I_{sp}$       | specific impulse                   | 7                             | combustor exit                           |
| $V$            | velocity                           |                               |                                          |

Received: 2008-04-22; Revision received date: 2008-06-30

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Introduction

A schematic illustration of solid propellant air-turborocket (SPATR) is shown in Fig. 1. As the figure shows, the turbine is driven solely by the fuel-rich gases with high temperature and high pressure which result from fuel-rich solid propellant at gas generator, and compressor, which raises the pressure of the air, is driven by the turbine. These fuel-rich gases at turbine exit mix with the compressor discharge air and burn in a combustor. The hot gases enter the nozzle and exhausted with high velocity finally in order to produce the thrust. SPATR, which is a combination of traditional Solid propellant rocket engine and air-breathing turbine engine, offers an array of characteristics, including a wide range of flight Mach number(0~ 4.3) and altitude (0~ 30km), high thrust-weight ratio (20~ 40), specific thrust (1 500~ 2 000 N/( kg/s) ), thrust-frontal area ratio(5 000~ 50 000 N/ m<sup>2</sup>) and high specific impulse (6 000~ 12 000 N/( kg/s) )<sup>[14]</sup>.

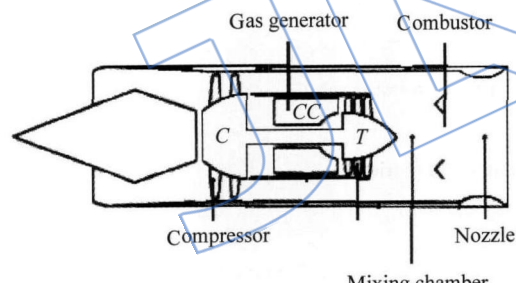


Fig. 1 Schematic illustration of solid propellant air turborocket ( SPATR)

A schematic illustration of operational modes for tactical missile equipped with SPATR is given in Fig. 2.

Compared with ramjet, a booster engine is not a must since SPATR has ability to accelerate itself. Apart from it, specific impulse of SPATR is higher than that of a rocket engine<sup>[5]</sup>. Therefore, SPATR is a promising power plant for tactical missile. During the development of computational model for air-turborocket (ATR), a thermodynamic model of ATR has been proposed by K. L. Christensen<sup>[67]</sup>. Combustor fuel-

air ratio ( $f$ ) was selected as a cycle parameter of thermodynamics in his model and thrust and specific impulse performance was analyzed in detail based on the  $f$ . However, the quantitative analysis of SPATR performance by component parameters is insufficient in his paper. In this paper, the quantitative analysis method is adopted to study the change in SPATR performance resulted from the change of component parameters, such as compressor pressure ratio and gas generator exit parameters. The off-design performance of SPATR has been computed and analyzed in detail. The performances of thrust and specific impulse for SPATR with different solid propellant have been compared at off-design points, and off-design performance comparison has been made between fuel-rich and oxygen-rich. The computation results indicated that SPATR meets the requirements of tactical missile.

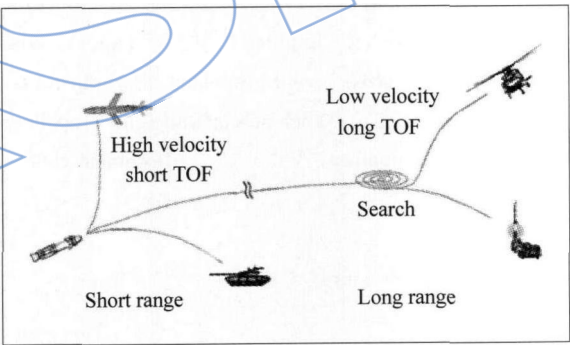


Fig. 2 Schematic illustration of operational modes for tactical missile equipped with SPATR

1 Thermodynamic model of SPATR

The balance equations that must be met for a thermodynamic model of SPATR include: the power balance between turbine and compressor, static pressure balance at mixer inlet and mass flow balance at nozzle inlet. These equations may be listed respectively:

$L_C = L_T$  (1)

$p_{5H} = p_{5L}$  (2)

$W_g = W_a + W_F$  (3)

Where

$L_C = W_a C_p a T_{12} \left( e^C - 1 \right) / \eta_c$

$$L_T = W_F c_{pF} T_{t4} \left( 1 - e_T^{-1} \right) \eta_T$$
$$e_c = \pi_c^{(\kappa_a - 1)/\kappa_a}, e_T = \pi_T^{(\kappa_F - 1)/\kappa_F}$$

According to the computation method in reference<sup>[8]</sup>, thrust, specific thrust and specific impulse may be listed respectively:

$$F_n = V_9 W_g + (p_9 - p_0) A_9 - V_0 W_a$$
$$F_s = \frac{V_9 W_g + (p_9 - p_0) A_9 - V_0 W_a}{W_a}$$
$$I_{sp} = \frac{F_n}{W_F} = \frac{1+f}{f} V_9 + \frac{(p_9 - p_0) A_9}{W_F} - \frac{1}{f} V_0 \tag{4}$$

The computation method of off design performance of SPATR is very similar to that of turbojet engine. It is selected as control law that the Shaft speed equals a certain constant ( $n = \text{const}$ ) and compressor pressure ratio ( $\pi_c$ ), turbine inlet total temperature ( $T_{t4}$ ) and fuel-rich gases flow ( $W_F$ ) are regarded as independent variables, the three balance equations (1, 2, 3) will form a nonlinear equations, as described as follow:

$$\begin{cases} E_1 = L_T - L_C = E_1(\pi_c, T_{t4}, W_F) = 0 \\ E_2 = p_{5H} - p_{5L} = E_2(\pi_c, T_{t4}, W_F) = 0 \\ E_3 = W_g - W_a - W_F = E_3(\pi_c, T_{t4}, W_F) = 0 \end{cases} \tag{5}$$

The nonlinear equations are solved by the multivariable Newton-Raphson method and the off-design performance of SPATR may be obtained in terms of the equations (5) after the desired tolerance on convergence is met.

Note that the fuel-rich gases from the turbine are irrelevant to the air from the compressor before being mixed, so the ratio of fuel-rich gases to the air in the combustor is regarded as a fuel-air ratio (i. e. bypass ratio). According to equation (1), the fuel-air ratio is derived as follows:

$$f = \frac{W_F}{W_a} = \frac{c_{pa} T_{t2}}{c_{pF} T_{t4}} \left( \frac{e_c - 1}{1 - e_T^{-1}} \right) \frac{\eta_c}{\eta_T} \tag{6}$$

As the equation (6) showed, combustor fuel-air ratio of SPATR not only gives the ratio of fuel-rich gases to the air, but also presents a functional relationship between the compressor pressure ratio ( $\pi_c$ ) and turbine pressure ratio ( $\pi_T$ ). In terms of the equation (4), fuel-air ratio

has an important influence on the thrust and specific impulse. If the propellant, oxidant and mixture ratio between them are selected, the proper mixing ratio ( $L_o$ ) which meets the need of fuel-rich in theory will be obtained by the empirical formula and fuel-air ratio is an equivalent fuel-air ratio in the combustor, namely,

$$f_o = \frac{1}{\alpha L_o} \tag{7}$$

Where  $\alpha$  is the residual air coefficient. Given  $f > f_o$ , combustor will operate under the condition of fuel-rich, as discussed in detail at the performance analysis of off design points.

## 2 Thermodynamic cycle analysis at design point

For the thermodynamic cycle analysis at design point, the influence of SPATR performance caused by the total pressure, total temperature and flow of gas generator exit are analyzed in detail. Table 1 presents the design point parameters of SPATR.

Table 1 Design point parameters of SPATR

| Design point parameters            | $H = 0, Ma = 0$ |
|------------------------------------|-----------------|
| Compressor pressure ratio          | 4.0             |
| Compressor efficiency              | 0.85            |
| Turbine pressure ratio             | 4.5             |
| Turbine efficiency                 | 0.85            |
| Turbine inlet total temperature/ K | 1100            |
| Shaft speed / ( % )                | 100             |
| Air flow/ ( kg/s )                 | 1.0             |
| Combustor efficiency               | 0.75            |

Fig.3 shows that the variation of fuel-rich gases ( $W_F$ ) at the gas generator exit has an obvious influence on the SPATR performance when air flow of engine remains a certain constant. As the figure shows, with the increase of fuel-rich gases ( $W_F$ ), the thrust increases and specific impulse reduces at the same time under the condition of  $f < f_o$ . However, combustor operates under the condition of fuel-rich given  $f > f_o$ , and

the increase of thrust is determined by the equivalent fuel-air ratio of combustor, but the decrease of specific impulse is computed based on equation (4). The change of thrust, which fuel rich gases flow equals 0.16 kg/s, reflects the variation from oxygen rich to fuel rich. Considering the compromise between thrust and specific impulse,  $W_F$  at design point is appreciably less than the one which the corresponding fuel-air ratio equals equivalent fuel-air ratio in order to produce the preferable thrust (and, specific thrust).

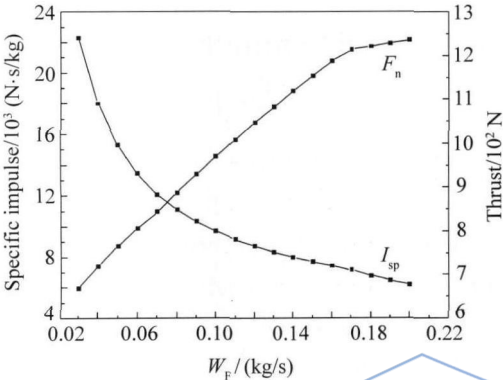


Fig.3 Influence on the SPATR performance caused by variation of fuel rich gases ( $W_F$ ) at the gas generator exit

Fig.4 illustrates the variation of total pressure ( $p_{14}$ ) and total temperature ( $T_{14}$ ) at gas generator exit has an important influence on the

SPATR performance when both air flow of engine ( $W_a$ ) and fuel-rich gases flow ( $W_F$ ) are a certain constant. With the increase of  $p_{14}$ , combustor total pressure and nozzle inlet total pressure increase respectively, which leads to the increase of exhaust velocity in order to produce the preferable thrust. Since the total pressure of combustor increases, compressor pressure ratio ( $\pi_c$ ) is inevitably rising properly to ensure the static pressure balance at turbine exit. However, the increase of  $\pi_c$  is needed to raise  $\pi_T$  to offer sufficient power to drive the compressor, and the increase of  $\pi_T$  leads to the decrease of combustor total pressure. Given that shaft speed equals a certain constant according to equation (1). Considering that  $f$  usually has a range from 0.08 to 0.2 kg/s and  $W_a$  is greater than  $W_F$ , the gain of  $\pi_T$  has to be bigger than that of  $\pi_c$  to ensure the power balance, as a result, combustor total pressure increases at the beginning, and then reduces; thrust and specific impulse follows the same pattern. Therefore the optimal  $p_{14}$  (see Fig. 4(a)) which ensures  $F_n$  and  $I_{sp}$  as maximums at the same time existed given that  $W_a$  and  $W_F$  are selected as design point parameters. The optimal  $p_{14}$  is used as a gist to select turbine pressure ratio at design point based on the constraint equations (1, 2).

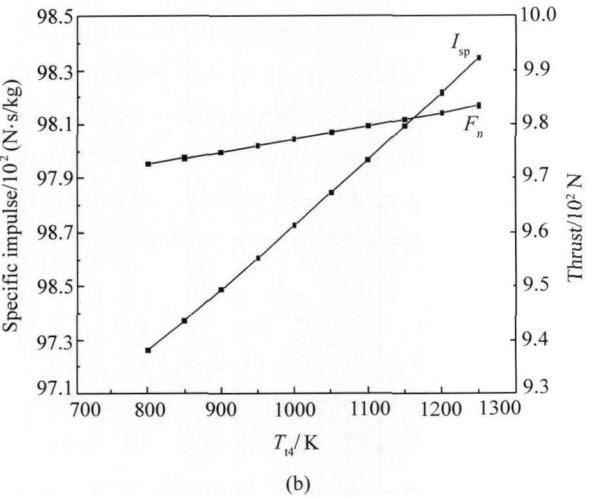
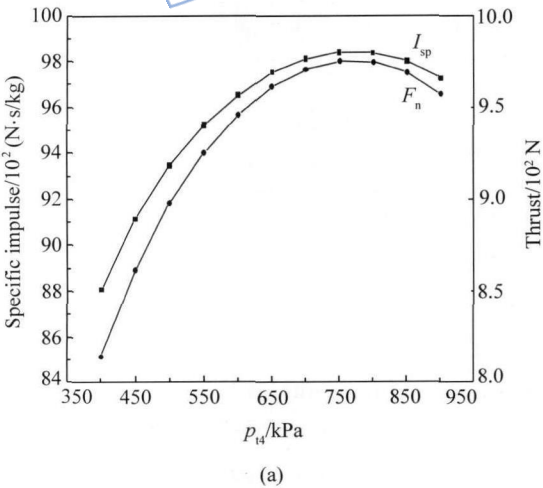


Fig.4 Influence on the SPATR performance caused by the total pressure and total temperature at gas generator exit at design point

Fig. 4(b) gives that the variation of total temperature ( $T_{14}$ ) at gas generator exit has an important influence on the SPATR performance. The increase of  $T_{14}$  means the increase of energy from solid propellant in the gas generator and decrease of energy from solid propellant in the combustor based on the energy analysis method. When combustor efficiency at design point equals a certain constant, the increase of  $T_{14}$  is useful to increase thrust and specific impulse, but the gain is not obvious. Note that the parameters determining the turbine power also include thermal parameters of fuel-rich gas, such as specific heat at constant pressure ( $c_{pF}$ ) and ratio of specific heats ( $k_F$ ).  $W_F$  is only related to  $c_{pF}$  and  $k_F$  when both turbine pressure ratio and  $T_{14}$  equals constants. The combustor operates under the condition of fuel-rich because of the increase of  $W_F$  from low  $c_{pF}$  and  $k_F$ , which not only reduces the specific impulse of SPATR, but also the application of solid propellant to SPATR is greatly limited. Since higher  $T_{14}$  leads to a wide choice of types and mixture ratio for solid propellant, a higher  $T_{14}$  is selected in theory. However, fuel-rich gases of turbine inlet are irrelevant to the air of compressor inlet and  $p_{14}$  is bigger than  $p_{15II}$ , so how to cool the turbine is a great challenge, and temperature limit of turbine inlet is irrelevant to flight condition, it only de-

pends on the materials. Therefore the turbine inlet total temperature of SPATR is less than that of turbojet. Considering that the requirement of stability margin at design point, that  $T_{14} = 1100\text{K}$  was used in this calculation.

SPATR performance is also determined by the combination of turbine and compressor. According to equation (6),  $f$  is a function of  $\pi_r$  and  $\pi_c$ . Given that  $\pi_c$  is selected;  $f$  and  $\pi_r$  are not irrelevant any more. so the thermodynamic cycle analysis of  $\pi_r$  were not carried out since  $\pi_r$  is not an independent variable. Besides, according to equation (1), a higher turbine pressure ratio at design point is useful to the power balance, but this means that multistage turbine are adopted with the increase of volume, complexity and cost for turbine. Considering that tactical missile have strict limit for volume, complexity and cost, that turbine pressure ratio equals 4.5 and corresponding efficiency equals 0.85 were selected in this paper.

Fig. 5(a) shows that the variation of compressor pressure ratio ( $\pi_c$ ) at design point has an important influence on the SPATR performance when both  $W_a$  and  $W_F$  are constants. With the increase of  $\pi_c$ , combustor total pressure and nozzle inlet total pressure increase respectively, which leads to the increase of exhaust velocity in order to increase thrust and specific impulse if

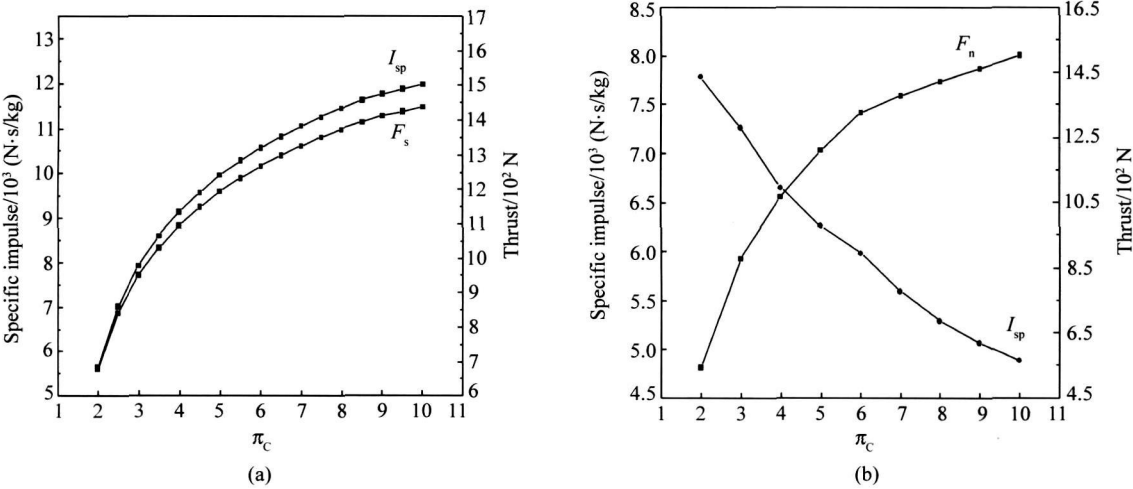


Fig. 5 Influence of on the SPATR performance caused by variation of compressor pressure ratio at design point

turbine pressure ratio meets the requirement of equations (1, 2). When  $\pi_T = \text{const}$  and  $W_F$  meets the change of equations (1), thrust increases and specific impulse reduces with the increase of  $W_F$ . Compressor pressure ratio at design point is selected according to the trend in Fig. 5(b). Considering the compromise between thrust and specific impulse, a crosspoint of thrust curve and specific impulse curve, which  $\pi_c$  equals 4.0, was selected as a design point. The compressor pressure ratio in this paper was determined by this method.

### 3 Performance analysis at off design points of SPATR

According to thermodynamic cycle analysis at design point of SPATR, the parameters at design point are determined preliminary. These parameters lists in table 1 are used as the initial values to compute the off design performance of SPATR.

Note that the compressor inlet total temperature ( $T_{12} > 800$  K) would surpass the allowable value which is determined by compressor usually made of Aluminum Alloy if  $Ma > 3.0$  at sea level. However, compressor inlet temperature limit is not taken into account since thermodynamic cycle analysis of SPATR is only computed in theory in this paper. Fig. 6(a) gives the off design performance of SPATR when  $Ma = 0$  and  $H = 0$  m. Given that Shaft speed equals a certain

constant ( $n = \text{const}$ ) is selected as control law, with the increase of Mach number, gas flow and air flow of turbine inlet increases monotonously, and thrust reduces slightly when  $Ma < 0.5$ , but it increases monotonously when  $Ma > 0.5$ . On the other hand, with the increase of Mach number, specific impulse reduces rapidly when  $Ma < 0.5$ , then it will increase to some extent. However, specific impulse reduces rapidly when  $Ma > 2.0$  because nozzle exit area ( $A_9$ ) is a certain constant. When  $Ma > 3.0$ , computation is ceases since  $\pi_c$  reduces slightly. Fig. 6(b) presents the off design performance of SPATR when  $Ma = 0.8$  and  $H = 12$  km. As shows in Fig 6, the trend in change of (a) and (b) is similar. However,  $T_2$  and corresponding  $W_F$  are reduced with the rise of flight altitude. Besides, specific impulse at high altitude is bigger than that at sea level, but thrust at high altitude is less than that at sea level. Obviously, the computation was carried out while Mach number equals 4.0 since  $p_{12}$  is less than that at sea level.

The performance comparison of thrust and specific impulse for SPATR has been made between fuel-rich and fuel-poor in Fig. 7. Fuel-air ratio of fuel-poor and fuel-rich is given by 0.087 and 0.173 kg/s respectively. When Mach number is a certain constant, thrust of fuel-rich is more than that of oxygen-rich, but specific impulse of fuel-rich is less than that of oxygen-rich. Actually, in order to obtain sufficient thrust to

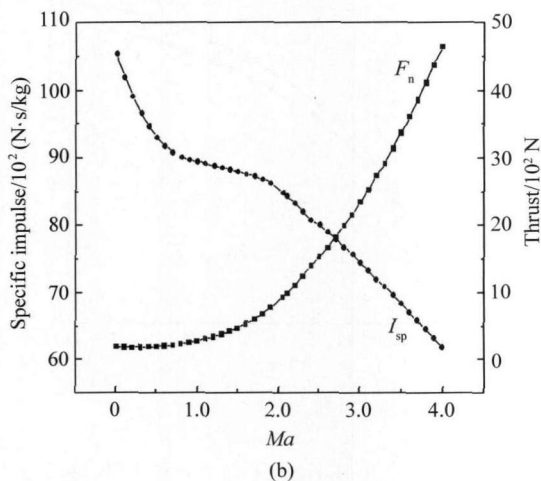
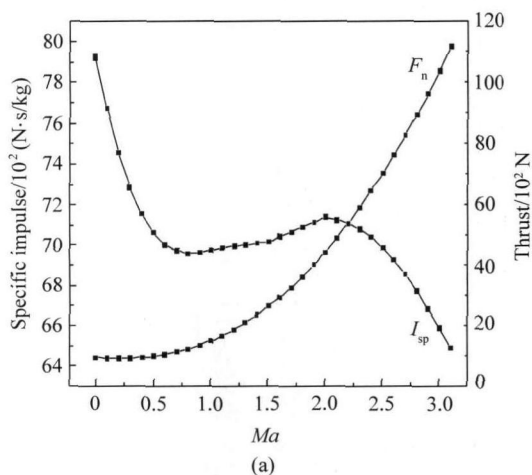


Fig. 6 Off design performance of SPATR

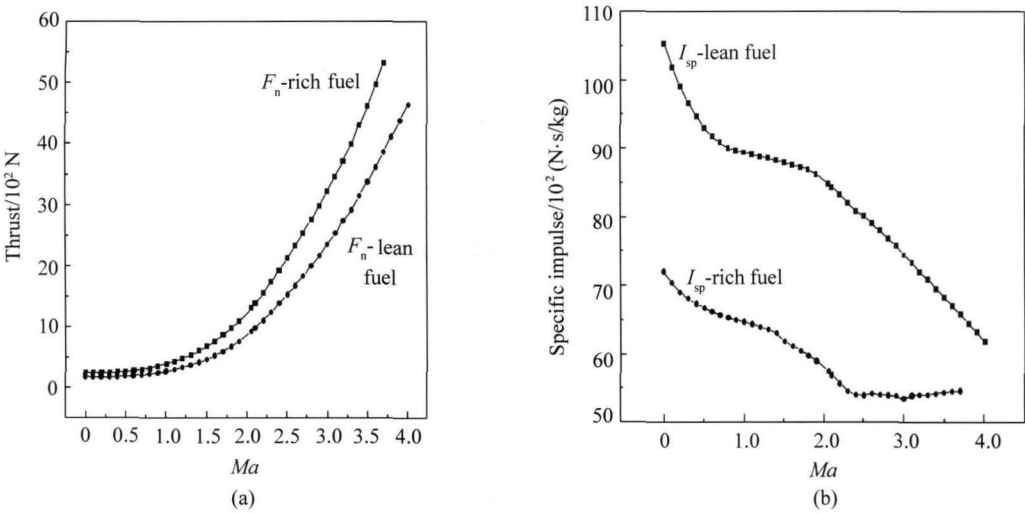


Fig. 7 Off design performance comparison of SPATR between fuel rich and oxygen rich

meet the requirement of flight, the combustor of SPATR usually operates under the condition of fuel-rich, which is different from turbine engine. Nevertheless, specific impulse of SPATR is still more than that of rocket since its specific impulse is only  $3\,000 \text{ N/(kg}\cdot\text{s)}$ .

The off design performance comparison of for SPATR with different solid propellant presents in Fig. 8. AP-based fuel-rich propellants were select as solid propellant and their thermal parameters were listed in Table 2.

Table 2 Thermal parameters of solid propellant

| Parameters                                | Propellant 1 | Propellant 2 |
|-------------------------------------------|--------------|--------------|
| $H_v/(\text{MJ/kg})$                      | 25.81        | 20.14        |
| $c_p/(\text{J}/(\text{kg}\cdot\text{K}))$ | 2180.0       | 1863.0       |
| $K$                                       | 1.1142       | 1.2168       |
| $L$                                       | 5.251        | 5.998        |
| $R/(\text{J/kg}\cdot\text{K})$            | 378.92       | 374.02       |
| $f$                                       | 0.1904       | 0.1667       |

Given that design point parameters of SPATR are constants on the same flight condition, the total energy of combustor is determined only by the types of solid propellant according to the equation listed as follows:

$$H_{17} = H_{16} + \frac{H_v \cdot W_a \cdot f \cdot \eta_B}{W_g} = H_{16} + \frac{H_v \cdot f \cdot \eta_B}{1 + f} \tag{8}$$

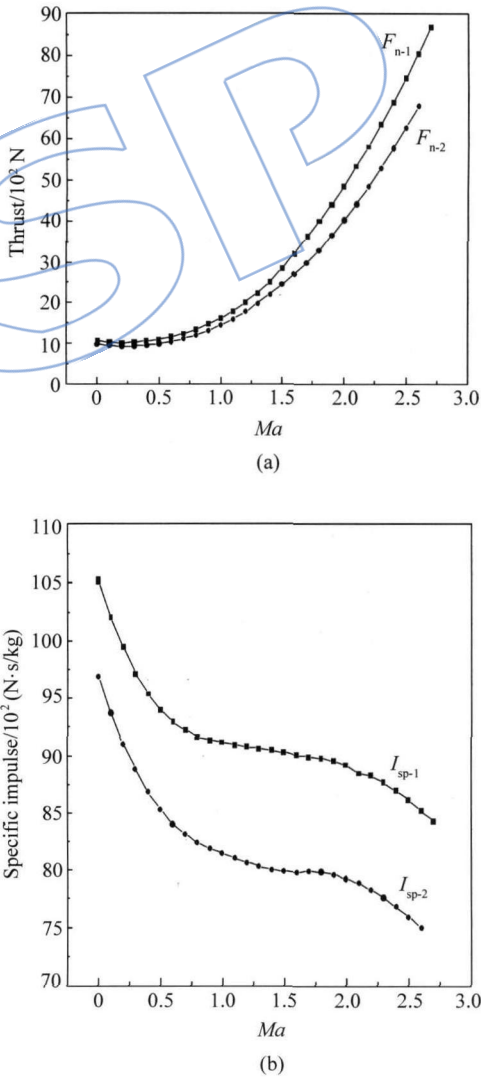


Fig. 8 Off design performance comparison of SPATR with different solid propellant

As the equation shows, total energy of combustor, thrust and specific impulse of SPATR are determined by low heat value of solid propellant when mixture ratio of propellant and oxidant is given in advance. Since thrust and specific impulse is directly in proportion to the low heat value, solid propellant with a higher low heat value was selected to obtain a better performance.

SPATR operates within a wide range of Mach number (0~3) and altitude (0~12 km) according to the above computation results and analysis of design point and off-design points. Once fuel/air ratio of combustor is an equivalent fuel/air ratio, specific thrust has reached 1 200 N/(kg/s) and specific impulse also has been 7 000 N/(kg/s) if solid propellant-2 is adopted. Even though combustor operates under the condition of fuel-rich, specific impulse has reached 5 000 N/(kg/s) (see Fig. 7(b)), which is more than that of a generic rocket engine. Note that medium-energy solid propellant whose low heat value is only within the range from 18 to 25 MJ/kg is adopted in the above calculation. Actually, thrust and specific impulse would be increased rapidly, if high-energy solid propellant whose low heat value is more than 40 MJ/kg is applied to SPATR.

## 4 Conclusions

The performance of SPATR is determined by solid propellant and performance of components of engine. On the basis of technologies of solid rocket engine and turbine engine in existence, the cycle parameters at design point and the parameters of components for SPATR are selected reasonably in order to obtain the preferable thrust and specific impulse. The computation results of off-design

points for SPATR indicated the selection of parameters is feasible and rational.

Given that Shaft speed equals a certain constant ( $n = \text{const}$ ) is selected as control law, solid propellant that produces the fuel-rich gases with preferable  $c_{pF}$  and  $k_F$ , high turbine inlet temperature, high turbine pressure ratio and efficiency are chosen in order to increase the specific impulse of SPATR at high Mach number or avoid the rich-oxygen at low Mach number in this paper.

The improvement of solid propellant has an important influence on performance of SPATR. Actually solid propellant with high low heat value achieves the preferable thrust and specific impulse of SPATR.

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